Journal of Orthodontics/Vol. 28/2001/29–38

Scientific Section

A Validated Finite Element Method Study of Orthodontic Tooth Movement in the Human Subject

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Abstract. *The aim of the study was to develop a 3D computer model of the movement of a maxillary incisor tooth when subjected to an orthodontic load. A novel method was to be developed to directly and accurately measure orthodontic tooth movement in a group of human volunteers. This was to be used to validate the finite element-based computer model.*

The design took the form of a prospective experiment at a laboratory at the University of Wales in 1996/7.

A laser apparatus was used to sample tooth movement every 0·01 seconds over a 1-minute cycle for 10 healthy volunteers, whilst a constant 0·39 N load was applied. This process was repeated on eight separate occasions and the most consistent five readings taken for each subject. Data were used to calculate the physical properties of the periodontal ligament (PDL). The data gleaned by this method were used to validate the 3D FEM model. This was formed of 15,000 four-noded tetrahedral elements.

Tooth displacements ranged from 0·012 to 0·133 mm. An appropriate elastic modulus of 1 N/mm^2 and Poisson's Ratio of 0·45 was derived for the PDL. Strain analysis, using the model, suggested that a maximum PDL strain of 4·77 $\times$ 10^{-3} was recorded at the alveolar crest, while the largest apical strain recorded was 1·55 $\times$ 10^{-3}. The maximum strains recorded in the surrounding alveolar bone were 35 times less than for the PDL.

A novel method for direct measurement of PDL physical properties in the human subject has been developed. The validated FEM model lends further evidence that the PDL is the main mediator of orthodontic tooth movement.

Index words: Experimental Validation, FEM modelling, Laser Measurement, Periodontal Ligament Properties, Tooth Movement.

Introduction

It is now nearly two thousand years since the phenomenon of tooth movement in response to an applied load was first reported (Celsus, 1st century AD). Currently, although teeth are moved routinely in orthodontic practice, it is still the case that there is much to learn about the exact ongoing changes in the biomechanical loading of tissues and the precise mechanism of tissue response following force application to the crown of a tooth. The remodelling of the load-bearing tissues within the human body has been considered for many years to be influenced by the loads they carry. This reaction is well known to clinicians; however, the association between loading and structure has proved difficult to quantify. A number of early investigators attempted to relate tooth movement to the applied force, developing theories based on very simple and, inevitably, imprecise experimental techniques on human subjects (Storey and Smith, 1952; Lee, 1965).

Most of the experimental work performed in the area since that time has been based on animal experimentation (Sandstedt, 1901; Reitan, 1957; Picton & Davies, 1967; Rygh, 1974, 1977; Davidovitch and Shanfield, 1975; Wills *et al.*, 1972; Chiba and Ohkawa, 1980; Yamaski, 1983; Hong, 1990); this work has been extensively reviewed previously (Hickman, 1997). Arguably, this type of approach can only give a crude indication of the likely biomechanical consequences in the human, since animal tissues, in this instance, are often poor morphological and biomechanical reflections

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0301-228X/01/010000+00$02.00

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of the matching human tissues. More recently, other authors have developed tissue culture systems to examine the effects of stress on osteoblast cells (Sandy, 1993). There is no doubt that the examination of the effects of stress at the cellular level and the attempts at understanding the mechanisms by which cell reactions are stimulated constitutes a fast-moving and fascinating area of research, extensively reviewed in a series of papers over the last decade (Sandy and Farndale, 1991; Sandy *et al*., 1993; Sandy, 1998). Although of considerable interest, this approach can only begin to reflect the complex stress/strain fields involved. Therefore, it is also important to gain an improved biomechanical understanding of what occurs 'further up the chain of events' immediately after the application of a continuous load on a tooth crown. In time, the linking of these areas of research will provide a much clearer picture to the clinician of what happens from the initial (macroscopic) application of an orthodontic force all the way down to the cellular response to the resultant local strains.

In an attempt to better understand the biomechanics of tooth movement, a variety of methods have been used to try to 'predict' tissue response to load. These have included theoretical mathematical techniques (Steyn *et al*., 1978), photo-elastic systems (Caputo *et al*., 1974), and laser holographic interferometry (Burstone and Pryputniewicz, 1980). However, such techniques have the disadvantage of only examining surface stress, whilst having the added problem of usually being supported by poor validation systems, as judged by current standards.

In the last decade the application of a well proven predictive technique, originally used in structural analysis, the finite element method (FEM) has revolutionized dental biomechanical research. The technique has been described, in detail, in an early standard text by Zienkiewicz (1977). Basically, the object to be studied is graphically simulated in a computer in the form of a *mesh,* which defines the geometry of the body being studied. This mesh is divided, by a process known as *discretization*, into a number of sub-units termed *elements*. These are connected at a finite number of points called *nodes*, which are, in turn, defined by their global co-ordinates. The constituent elements are prescribed the appropriate material properties of the structure they represent. What is achieved is a mathematical model of the likely physical response of that object to load; large volumes of information on stresses, strains, and displacements being obtained through the continuum defined.

Early work in this area in orthodontics focused on the development of crude 2D models using existing information on the physical properties of dry/wet bone and other tissues. Inevitably, the validation systems were very limited in scope (Williams and Edmondson, 1984; Williams *et al*., 1986). Since that time, three-dimensional FEM models of the tooth, periodontal ligament, and bone continuum have been described, a recent example being the work of Nyashin and co-workers (1997). Simple time-dependency and visco-elastic properties have also been introduced to make these models more useful in the theoretical analysis of the tissue reaction to orthodontic load (McGuinness *et al*., 1992; Wilson *et al*., 1992), whilst giving the opportunity to examine the important, but often neglected area of tissue strain. Such predictive models have, on occasion, been found to reflect existing, historical experimental data on tooth displacement following load (Ross *et al*., 1976), although good useful information is sparse and the methodology employed in collection can often be questioned. However, in any model simulating 'real-life' behaviour, an appropriate validation method is essential, to be confident of the accuracy of the results.

Some workers in this area of biomechanics have approached the problem by developing computational models of tooth movement in animals, using well recognized animal experimental techniques for validation (Tanne *et al*., 1987). However, the current study describes a different approach whereby a new technique of direct measurement of tooth movement in humans is used to validate a finite element model. The principles, the process of development and initial results obtained have been detailed and described previously (Volp *et al*., 1996; Hickman, 1997, Hickman *et al*., 1998; Jones *et al*., 1999). Since these models are of the human subject, rather than of animals, the results, when interpreted, are far more likely to be of direct relevance to clinicians.

Therefore, the aims of this investigation are to:

(1) develop an accurate and validated 3D FEM computer model of upper central incisor tooth movement in a typical human subject;
(2) have an appropriate model available to study the behaviour of teeth and surrounding tissues under load over a period of time.

The initial objectives are to:

1. Establish the early response of teeth to load and confirm the material properties of the periodontal ligament (PDL) through direct measurement on human volunteers.
2. Develop a 3D-computer simulation of maxillary incisor tooth movement using the finite element method (FEM). Initially, historical and then experimentally derived data on the physical properties of the surrounding tissues to be used to develop this simulation.
3. Using this model, examine in detail, the stresses and strains in the surrounding tissues associated with the tooth movement and, in particular, the nature of the displacement of the periodontal ligament (PDL).

Materials and Methods

A novel experimental method-employing laser measuring equipment described, in detail, previously (Volp *et al*., 1996), was further developed and used in 10 human volunteers to test the *in vivo* tooth response to load over time. A constant force was applied to the tooth under test by a stainless steel ball-ended probe, which was adjustable in three dimensions. A piston, responding to air pressure and regulated by a pressure sensor, applied a load via the probe. The resultant tooth displacement was measured by a laser beam trained at a target mounted at right angles and in a constant relationship to the probe. The laser was a Class 2 LAS-501V product with a 1-MW maximum and a 680-nm wavelength. The displacements plotted via this apparatus were also to be used to determine the properties of the periodontal ligament. Initial calibration was performed against a material of known dimensions and elastic modulus (Volp *et al*., 1996).

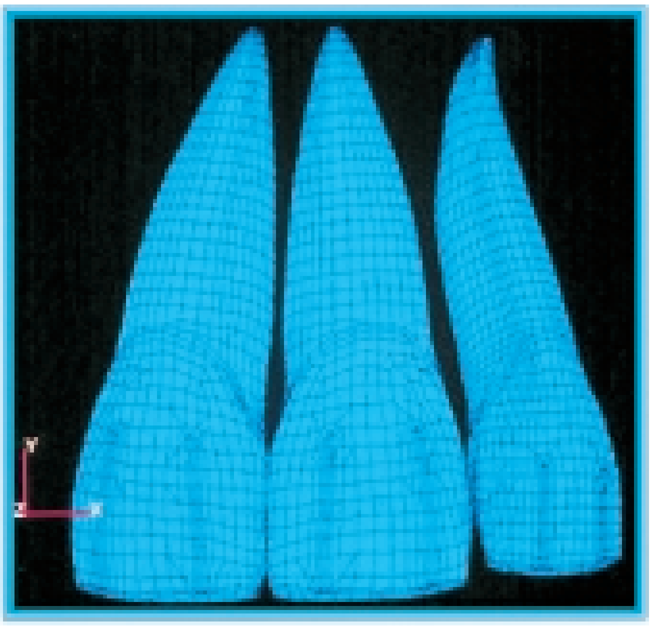
FIG. 1 (a) Novel apparatus for the loading and measurement of tooth displacement. Note the universal joint mounting of the assembly, which is counterweighted and allows the apparatus to move with the patient's head. (b) Detail of loading mechanism—laser target is also in view.

In the experimental phase of the study the loading system was adjusted to achieve a low, well-defined and precise force of 0·39 N on a maxillary incisor tooth. This force was of a continuous nature. Figure 1a shows the equipment in operation with one of the volunteers. Figure 1b shows the detail of the laser, target, and loading probe described above. Such an approach facilitates the detailed examination of time dependent change in tooth displacement, there being the capacity in the apparatus to sample every 1/100th of a second. The accuracy has been previously confirmed (Volp *et al.*, 1996) at 0·001 mm. Measurements in this current study were taken over a 1-minute cycle, with the first 10 seconds pre-load (to achieve a steady baseline reading to the cycle), then 30 seconds under a constant load, followed by a 20-second recovery phase.

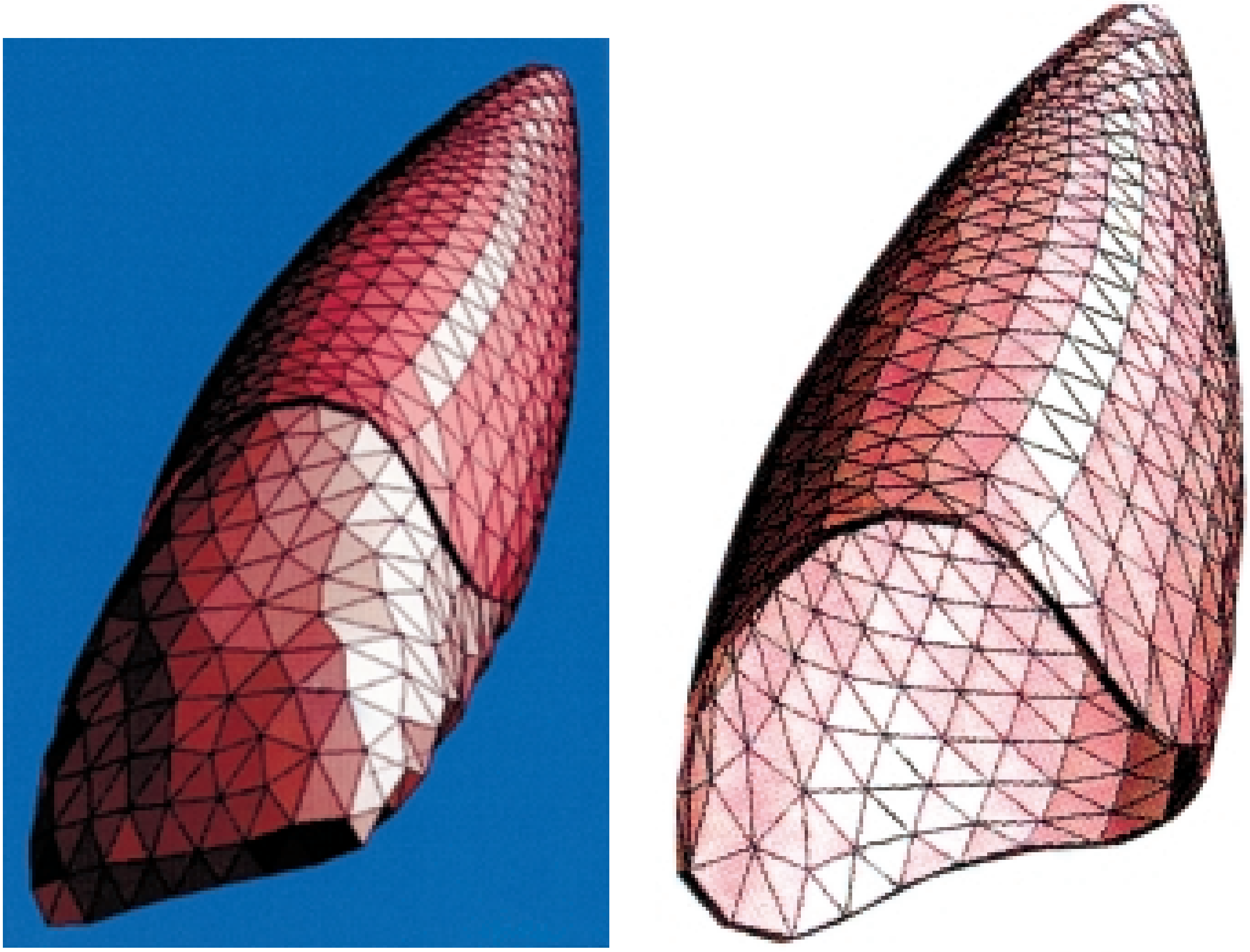
Detailed load displacement plots were obtained for 10 patients, judged to be typical of young healthy adults. (A number of subjects had been examined previously, and it was found that both age and periodontal health were important factors in the PDL response). The mean age of the subjects was 30·7 years with a range of 24·7–36·5 years. The subjects involved in the study had to have, on clinical examination, good soft-tissue health with a minimal depth of gingival crevice (less than 2 mm). The mesial and distal contacts were evaluated according to the ease of passage of dental floss and those with 'tight contacts' were excluded. The dimensions of each tooth were recorded (mean mesio-distal width 9·95 mm) and the mid-point of the labial surface of the crown was consistently located for the application of load. One difficulty with the measurement system was maintaining the point of force application and a method was developed to address this difficulty, which involved attaching a glass ionomer (GIC) cemented 'female marker' to the enamel surface of the incisor. The female marker consisted of an adapted 'Begg lingual button' with the surface ground then polished to give a concave hollow surface. This doubled as a locator, as well as acting as an electrical sensor of first probe contact. This received the rounded probe through which the load was applied.

The data collected was to be used to establish PDL behaviour under orthodontic load. To achieve a steady and more consistent displacement recording, volunteers chewed a standard material (chewing gum) for 5 minutes prior to the load of 0·39 N being placed on the incisor tooth to be measured. Successive tests on individuals were performed with a rest period of no less than 3 minutes and no more than 5 minutes. Ten volunteers were measured five times and showed significant inter- and intra-subject variability, although two patients were able to give a very consistent, reproducible reading.

The experimental results were used to assist in the derivation of an accurate three-dimensional finite element model. This was designed to reflect the anterior maxillary teeth and jaws of a typical human subject (Figure 2a,b,c). In parts of the model, routines were included for automatic adaptive mesh refinement and, since it was anticipated from previous work by the authors (Middleton *et al.*, 1996) that the PDL was particularly important in tooth movement, considerable effort was committed to modelling this area in detail.



(a)



(b) **(c)**

FIG. 2 (a) Basic morphology of mesh for teeth. (b,c) Detail of maxillary incisor tooth model showing tetrahedral elements forming the tooth and (shown separately) the periodontal ligament.

The finite element model was developed on a mainframe computer equipped with MSC/PATRAN software (MacNeal-Schwendler Co. Ltd, MSC House, Frimly, Surrey UK). The basic morphology of the mesh (Figure 2a) was formed using existing information via access to the male data from the 'Visible Human Project' (Viewpoint Data, 625S State, Orem, USA).

A mesh of 15,000 elements was constructed, the element used being of the four-noded linear tetrahedral type (figure 2b). This element was chosen since it is good at meshing arbitrary geometries—a prerequisite in this project. The tooth was divided into two basic materials: dentine and enamel. The surrounding alveolar bone (with compact and cancellous layers) and the periodontal ligament was also included in the model (Figure 2c). The basic initial material properties employed in the starting model are shown in Table 1.

Using the FEM model the predicted response to load was initially validated against previous data from earlier models and historical data (Ross *et al*., 1976; McGuinness *et al*., 1992; Wilson *et al*., 1992). Most importantly, it was then tested against the experimental results obtained in the parallel experimental study. This was achieved by adjusting the physical properties of the PDL in the FEM model until the computed displacement matched the experimental clinical results; thus the Elastic Modulus and Poisson's Ratio could be deduced.

Results

The results are summarized in Figures 3–5 and also in Table 2.

In Table 2 the series of total tooth displacements measured for the 10 volunteers ranged from 0·012 to 0·133 mm with a similarly wide range of variation in some subjects (4 and 6). Other subjects showed much better reproducibility (e.g. subjects 7 and 10).

TABLE 1 *Some of the historical material properties used in FEM model (after Wilson, 1992)*

Material	Youngs modulus (N/mm^2)	Poisson's Ratio
Enamel	84,100	0·2
Dentine	18,600	0·31
Cancellous bone	345	0·31
Cortical bone	13,800	0·26
Periodontal ligament	50	0·45

A typical plot of orthodontic incisor tooth movement is shown in Figure 3a, with an example of the possible reproducibility that can be obtained shown in Figure 3b.

Figure 4a–d show typical graphical plots of the strains produced in the parallel FEM computer model.

A typical recording for a maxillary central incisor displacement, measured in μm and calculated from the plots, is shown graphically in Figure 5a, whilst the typical centre of rotation is demonstrated in Figure 5b.

With reference to Figure 3a,b, these demonstrate the duration of time after loading for the initial visco-elastic phase on the plot before the displacement starts to plateau.

Discussion

The values for total displacement of the PDL over a one minute cycle for the 10 subjects ranged from 0·012–0·133 mm (or 133 μm) but showed coefficients of variation which ranged from 7 and 9 per cent in subjects 7 and 10, to 59 and 64 per cent in subjects 6 and 4. This was not an unexpected variation, since both age and disease are amongst a number of factors that might influence the properties of the PDL (Tanne *et al*., 1998). Nonetheless, it demonstrates that appropriate average values are best applied to this type of FEM simulation model. Having said this, reasonable consistency could be achieved in certain individuals—partly due to their ability to relax within the apparatus. The mean maximum displacement was 87·7 μm [SD (standard deviation) = 50·71]. The mean elastic modulus for a typical ligament was calculated for the PDL through identification of the elastic phase of the tooth displacement on the plots. This was applied to the FEM model. A typical plot of an incisor tooth under load is shown in Figure 3a. The consecutive plots, taken over two separate occasions and shown in Figure 3b, demonstrate the reproducibility of the tooth displacement measurements possible in some of the subjects.

The detail of the model is discussed elsewhere (Hickman, 1997), but it comprised of a basic 15,000 × 3-D tetrahedral elements. The tooth movement analysed on the model was found to basically follow rigid body motion with an initial instantaneous centre of rotation towards the apical third of the root (Figure 5b). The movement of the root of the tooth was comfortably within the PDL space

TABLE 2 *Maximum recorded tooth displacements on nine volunteers measured eight times (subject 8 was only measured on one occasion)*

Subject no.	Maximum displacement (μm)				Coefficient of variation (%)	Rank order
	Mean	Min	Max	S.D		
1	89·9	12	205	50·71	56	5
2	130·4	97	166	24·79	19	3
3	132·6	82	186	47·54	36	4
4	99·6	43	205	63·79	64	9
5	80	39	159	46·79	58	6
6	73	33	133	43·51	59	8
7	12·4	12	14	0·89	7	1
8	117	–	–	–	–	–
9	62·6	30	116	36·4	58	7
10	103·6	90	112	9·01	9	2
All	87·7	12	205	50·71	58	–

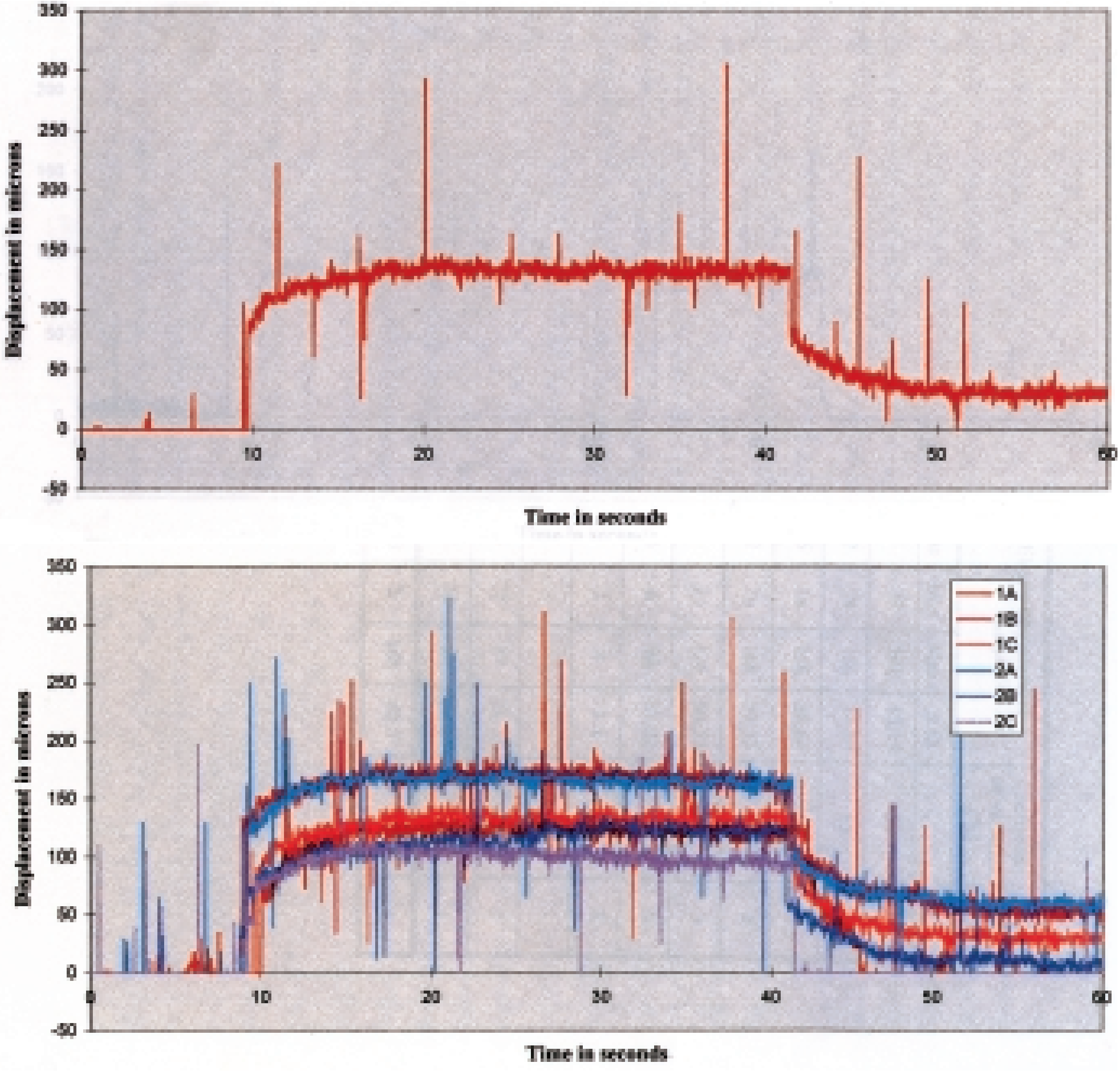


FIG. 3 (a) Plot showing typical profile of tooth displacement versus time. Note initial visco-elastic phase followed by plateau then recovery phase after removal of load. (b) Demonstrating reproducibility of displacement plots that can be achieved. Same load placed on six occasions at two separate occasions (subject 2).

and stresses were found to be within the maximum for physiological movement as suggested by Lee (1965). However, the stress/strain fields within the modelled PDL were found to be very complex.

The maximum displacement occurred at the cervical margin (the neck of tooth at the junction between crown and root). This is the area where one might expect greater stresses, leading to a potential for cell hyalinization within the ligament. Such stresses may also lead to local undermining resorption of the bone of the socket wall as part of a pathological response. Such a localized reaction can have a significant effect on the predictability of the tooth movement. In this area the strains noted were largely shear across the PDL.

The maximum principal strains in the periodontium were concentrated in two areas: at the alveolar crest on the buccal aspect and at the palatal side closest to the incisor root apex. The magnitude of these strains was greatest at the alveolar crest reaching a peak of $4{\cdot}77 \times 10^{-3}$, while the largest value of apical strain was $1{\cdot}55 \times 10^{-3}$. By comparison, the maximum principal strains seen in the alveolar bone most adjacent to the ligament were at least 35 times less ($1{\cdot}4 \times 10^{-4}$). Elsewhere in the bone, strains approached zero.

Interestingly, large strains were found to be localized to the periodontal ligament, but only negligible strains were found at the surface of the tooth root and bony socket. This data, together with that obtained from a previous, largely

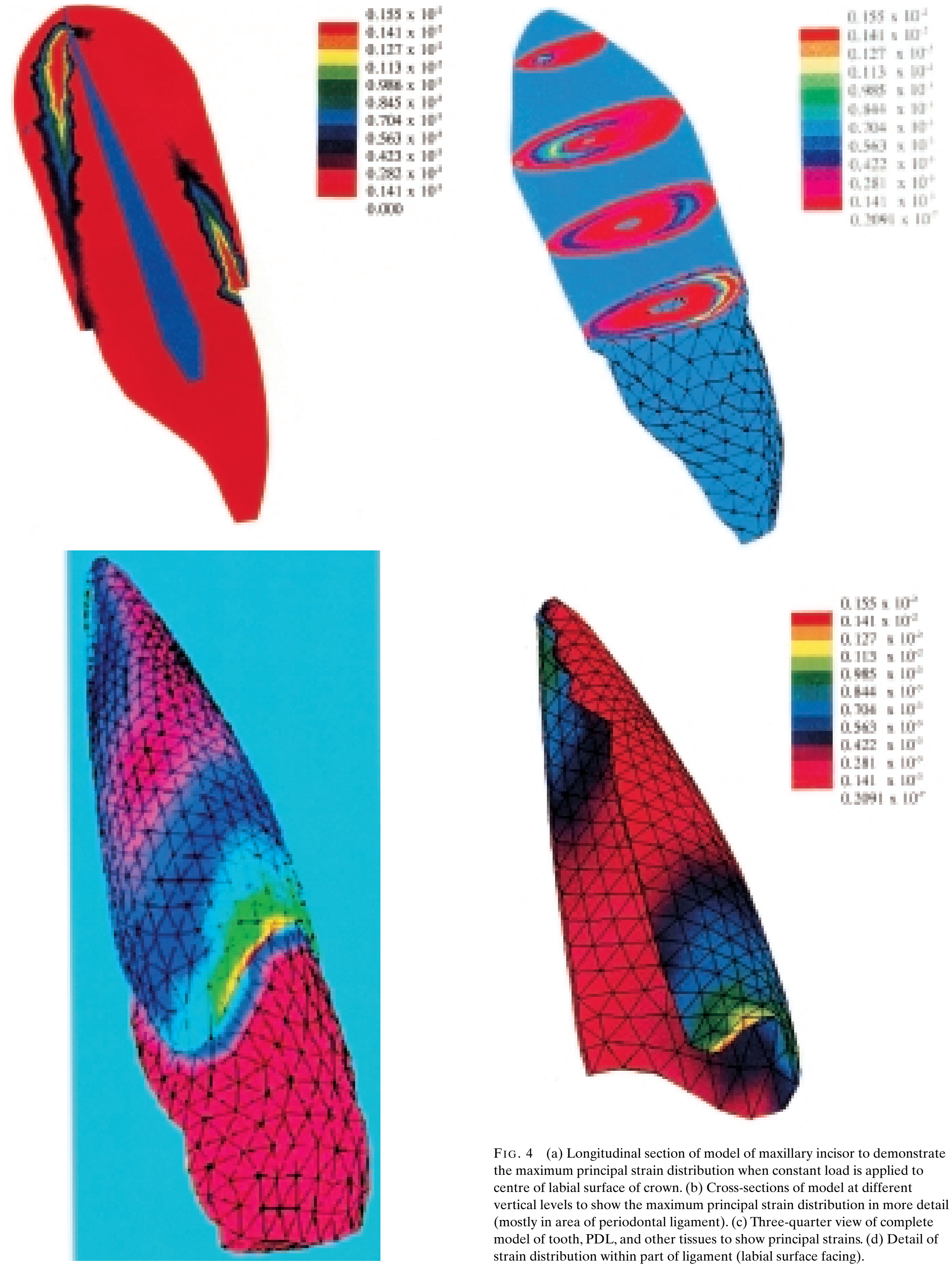


FIG. 4 (a) Longitudinal section of model of maxillary incisor to demonstrate the maximum principal strain distribution when constant load is applied to centre of labial surface of crown. (b) Cross-sections of model at different vertical levels to show the maximum principal strain distribution in more detail (mostly in area of periodontal ligament). (c) Three-quarter view of complete model of tooth, PDL, and other tissues to show principal strains. (d) Detail of strain distribution within part of ligament (labial surface facing).

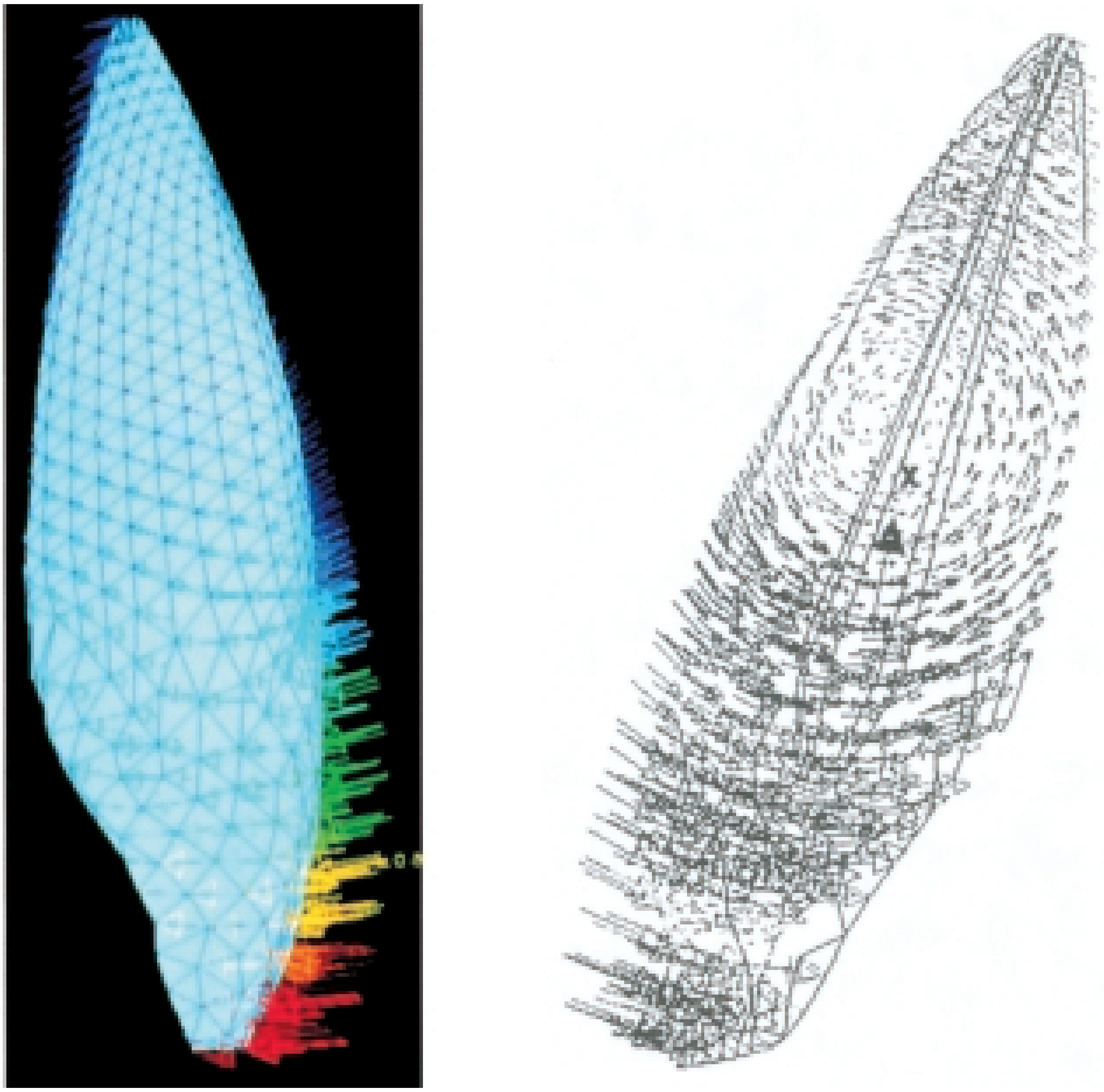


FIG. 5 (a) Typical recording of tooth displacement over time derived from original plots and transcribed to a 2D diagram. (b) Figure to show plotted motion of tooth demonstrating centre of rotation.

theoretical, model (Middleton *et al.*, 1996), suggests that initial orthodontic tooth movement must be largely mediated via the periodontal ligament, rather than by any cellular remodelling response originating in the local bone. Only in the PDL did the strains recorded exceed the minimum threshold previously suggested by Frost (1983) as being necessary to activate a local bone resorption process.

Finding an absolute value for the elastic modulus of the periodontal ligament based on the experimental data proved difficult and, in any case, was probably inappropriate, since there are such large variations between individuals. A good working assumption is that in a young adult with a healthy PDL, the elastic modulus is not likely to be less than 0·18 N/mm^2 and is most probably usually in the region of 1 N/mm^2.

This experimental work on human volunteers examining tooth displacements with the laser displacement apparatus has been established as a valid approach and the apparatus has provided a number of interesting findings. First, the initial elastic and then basic visco-elastic behaviour of the ligament has been confirmed, although it was observed that there is variability in the length of these initial phases of displacement in different individuals. Secondly, it is apparent that in order to determine the physical properties of the ligament (by back calculation) that the apparatus needed significant modification to be able to place an accurate light load over time. This gives a slower initial displacement, which results in a clearer picture of the early phase of ligament behaviour in response to typical orthodontic load.

The plots suggest that, even when allowing for some artifactual behaviour, it may take some time for the PDL to fully recover with the tooth returning to its pre-loaded position (i.e. zero). On a number of occasions the tooth hadn't 'zeroed' during the 1-minute cycle of this experiment—whether in these cases the tooth eventually returns to its start position is currently the subject of further investigation.

The type of predictive computer model described may be used to study the biomechanics of tooth movement, whilst accurately assessing the effect of new appliance systems and materials, without the need to go to animal or other less representative models. However, the ongoing challenge in this type of work is to confirm the accuracy of the numerical computer model. This will often require an exhaustive validation process—one example of which has been described in this study. With this proviso, computer models of various types will be used increasingly for fundamental biomechanics research in dentistry. They can also provide an ideal 'test-bed' for the research and development of new materials for use in the mouth (Middleton *et al.*, 2000; Knox *et al.*, 2000). This will, in the future, in orthodontics, reduce the need for the types of prolonged clinical trials reported previously (e.g. Evans *et al.*, 1998).

Conclusions

1. A computer-based three-dimensional finite element model of a maxillary left incisor tooth together with its neighbouring teeth and surrounding tissues was created. A novel laser displacement apparatus to measure the movement of teeth under a constant load on a series of human volunteers was constructed. It underwent extensive testing and modification to support this study. Detailed accuracy trials at low loading levels were performed.
2. Displacements from *in vivo* testing showed much variation between subjects, but greater consistency within some subjects.
3. The PDL demonstrates an initial elastic response followed by a visco-elastic phase when subjected to a continuous load.
4. The experimental results acquired from this apparatus were fed into the finite element model and, thus, the computer simulation of tooth movement was validated.
5. The material properties of the periodontal ligament, a notoriously difficult material to quantify, were calculated from the experimental data.
6. An appropriate estimate of the Elastic Modulus of the PDL is 1 N/mm^2, whilst the Poisson's Ration is 0·45 as confirmed by the experimental results.
7. An important incidental finding was an inter-subject variability of response, which one might speculate to be related, in part, to an early disease process of the ligament. The apparatus described could form the basis of a non-invasive early detection mechanism for this common disease process.
8. The FEM model demonstrated that only the PDL was subject to any strain levels of significance. This supports the contention that the periodontal ligament is central to the tissue response to load and subsequent tooth movement.
9. This initial FEM computer model has demonstrated that such an approach can be valid in the detailed study of orthodontic biomechanics. The movement of a tooth was charted, validated, and the centre of rotation was described in detail.

Acknowledgements

The authors gratefully acknowledge the support for this research that came as a grant from the Dr Hadwen Trust for Humanity in Research.

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